USN ☐☐☐☐☐☐☐☐☐☐   17MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1   a. Find the modulus and amplitude of
$$\frac{3+i}{2+i}$$
(07 Marks)

   b. If $x = \cos\theta + i\sin\theta$, then show that $\frac{x^{2n}-1}{x^{2n}+1} = i\tan n\theta$.   (07 Marks)

   c. Simplify $\dfrac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$ (06 Marks)

OR

2   a. Find the sine of the angle between $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$.   (07 Marks)

   b. Find the value of λ, so that the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{k}$
are coplanar.   (07 Marks)

   c. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$.   (06 Marks)

Module-2

3   a. Find the nth derivative of $e^{ax}\cos(bx + c)$.   (07 Marks)

   b. If $y = a\cos(\log x) + b\sin(\log x)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.   (07 Marks)

   c. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.   (06 Marks)

OR

4   a. Find the pedal equation of $r^n = a^n \cos n\theta$.   (07 Marks)

   b. Expand $\log_e(1 + x)$ in ascending powers of x as for as the term containing x^4.   (07 Marks)

   c. If $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ (06 Marks)

Module-3

5   a. Evaluate $\displaystyle\int_0^1 \int_{y^2}^{y} (1 + xy^2)\,dx\,dy$ (07 Marks)

   b. Evaluate $\displaystyle\int_0^{2\pi} \sin^4 x \cos^6 x \,dx$ (07 Marks)

   c. Evaluate $\displaystyle\int_0^2 \frac{x^4}{\sqrt{4-x^2}}\,dx$ (06 Marks)

OR

6   a.   Evaluate $\int\limits_{1}^{2}\int\limits_{3}^{4}(xy + e^{y})\,dydx$ **(07 Marks)**

     b.   Evaluate $\int\limits_{0}^{\pi} x\sin^{8}x\,dx$ **(07 Marks)**

     c.   Evaluate $\int\limits_{1}^{2}\int\limits_{0}^{1}\int\limits_{-1}^{1}(x^{2} + y^{2} + z^{2})\,dxdydz$ **(06 Marks)**

Module-4

7   a.   If particle moves on the curve $x = 2t^{2}$, $y = t^{2} - 4t$, $z = 3t - 5$ where t is the time. Find the velocity and acceleration at $t = 1$.   **(07 Marks)**

     b.   Find the angle between the tangents to the curve $\vec{r} = t^{2}\hat{i} + 2t\hat{j} - t^{3}\hat{k}$ at the point $t = \pm 1$.   **(07 Marks)**

     c.   If $\vec{F} = (3x^{2}y - z)\hat{i} + (xz^{3} + y^{4})\hat{j} - 2x^{3}z^{2}\hat{k}$ find grad(div $\vec{F}$) at $(2, -1, 0)$.   **(06 Marks)**

OR

8   a.   Find the directional derivative of $\phi = 4xz^{3} - 3x^{2}y^{2}z$ at $(2, -1, 2)$ along $2\hat{i} - 3\hat{j} + 6\hat{k}$ **(07 Marks)**

     b.   Find the unit normal to the surface $x^{2}y + 2xz = 4$ at $(2, -2, 3)$.   **(07 Marks)**

     c.   Show that $\vec{F} = (2xy^{2} + yz)\hat{i} + (2x^{2}y + xz + 2yz^{2})\hat{j} + (2y^{2}z + xy)\hat{k}$ is irrotational.   **(06 Marks)**

Module-5

9   a.   Solve $\dfrac{dy}{dx} = \sin(x + y)$ **(07 Marks)**

     b.   Solve $\dfrac{dy}{dx} + y\cot x = \cos x$ **(07 Marks)**

     c.   Solve $(x - y + 1)dy - (x + y - 1)dx = 0$ **(06 Marks)**

OR

10   a.   Solve $\left(1 + e^{x/4}\right)dx + e^{x/y}\left(1 - \dfrac{x}{y}\right)dy = 0$.   **(07 Marks)**

     b.   Solve $\left(x^{3}\cos^{2}y - x\sin 2y\right)dx = dy$.   **(07 Marks)**

     c.   Solve $\left(3x^{2}y^{4} + 2xy\right)dx + \left(2x^{3}y^{3} - x^{2}\right)dy = 0$ **(06 Marks)**

* * * * *